A Mathematical and Regression Analysis Models on the Effect of the Inflation rate of Tanzania on Rwanda's Economy: A comparative Analysis

Venuste Ntakirutimana [1], Gratien Twagirumukiza [1], Charline Uwilingiyimana[2]

[1,2]INES Ruhengeri, Faculty of Applied Fundamental Sciences, Department of Statistics Applied to Economy (SAE), Northern, Musanze, Rwanda

E-mail addresses: venustntaki@gmail.com, gratien@aims.edu.gh, ucharline@ines.ac.rw

Abstract: **It has been proved that the Economy of Rwanda rely mostly on neighboring countries. In this study, Tanzania was considered as the most influencing Country on Rwanda's Gross Domestic Product (GDP) due to strong trade flow between the two neighbors.** *Objective*: **The study aim was looking at whether the Inflation rate of Tanzania has an influence on Rwanda GDP and/or not.** *Methodology*: **To look at this scenario, A Mathematical and Regression Analysis Models were used. We studied this scenario with real data of 23years, that is, 1996-2019.** *Results*: **Having used the aforementioned two Models, altogether as a comparative Analysis, give the same results. The findings show that the more inflation rate of Tanzania decreases, the more Rwanda GDP increases which shows a positive effect on Rwanda's Economy.** *Conclusion*: **Since the two models show a positive influence of Tanzania Inflation rate when it decreases, has on Rwanda's Economy, then it is predicted that if it goes high, the Economy of Rwanda would fall down.** *Recommendation*: **We recommend the government of Rwanda to strengthen trade flow with Tanzania and kept it forward.**

Keywords: **Mathematical Model, Threshold, Inflation rate, GDP, Rwanda-Tanzania.**

1. INTRODUCTION

Mathematical models have been used over a decade to deal with dynamism of things. The drift of Inflation and Gross Domestic Product % (GDP) between Rwanda and Tanzania, in the aforementioned context, can be illustrated using Mathematical Model and Statistics of the real data from 1996-2019. Inflation is defined as a sustained increase in the general price level of a broad range of products and services in a country over a lengthy period of time. Inflation, as defined by Hamilton (2001), is an economic condition in which the rise in money supply exceeds the new production of commodities and services in the same economy. Ojo (2000) and Melberg (1992) defined inflation as a general and consistent increase in the prices of goods and services in an economy. The percentage change in the price index is used to calculate the inflation rate (consumer price index, wholesale price index, producer price index etc).

Economic growth is defined as an increase in the size of a country's economy over time. Gross Domestic Product (GDP) is a broad measure of an economy's output. It's also known as the entire value of final goods and services produced within a country's boundaries in a given year, regardless of who owns them. This measure only accounts for final products and services, or those that are consumed by the end user rather than being utilized as a component of other goods Reddy (2012).

Ayyoub et al. (2011) conducted a study in Pakistan to assess the impact of inflation on GDP using the Ordinary Least Square (OLS) method on annual time series data from 1972 to 2009. According to the findings, the two factors exhibited a negative and substantial association. The study discovered, however, that the detrimental effects begin beyond a specific

amount known as the threshold level, which was set at 7%. Semuel (2014), on the other hand, looked at the impact of inflation, interest rates, and exchange rates on GDP in Indonesia and found that inflation had no meaningful impact.

Sarel (1996) investigates if inflation has a nonlinear effect on GDP and discovers a major structural break at an annual inflation rate of 8%. According to the findings, inflation has a minor beneficial influence on GDP below that amount but a strong negative effect on growth above that point. After establishing a joint panel database by gathering annual data from 87 nations for the period 1970-1990, the study used the OLS approach. In a study done in India from 1971 to 1998, Singh and Kalirajan (2003) found a negative association between inflation and GDP. They found that an increase in inflation has a negative influence on GDP.

The study by Bawa (2012) in Nigeria on the Threshold Effect of Inflation on Economic Growth using a threshold regression model. Results from this study revealed a negative relationship below and above the threshold value which was 13 percent but the effect was high above the threshold value compared to below. The studies by Fischer, (1993) and Barro (1995) on the relationship between GDP growth and macroeconomic variables like inflation and fiscal posture are other examples. For typical inflation rates above 10%, Barro finds a significant negative link between inflation and GDP growth.

Hansen (1999), Gonzalo, and Gonzalo (2002), take note of the non-linear link between inflation and economic growth, with inflation above a particular threshold assumed to have negative consequences on economic growth. Because the influence of inflation on economic growth is non-linear, low inflation is beneficial to growth while excessive inflation is negative. In Egypt, Hosny et al. (2014) estimated the Hansen threshold regression using data from 1981 to 2009. GDP growth, inflation, inflation squared, gross fixed capital formation, private sector credit, trade, openness, government consumption spending, nominal exchange rate, and population growth rate were all incorporated in their model. They concluded that Egypt's inflation threshold is 12.0%.

Fabayo and Ajilore (2006) used data from 1970 to 2003 to estimate a threshold regression model based on Khan and Senhadji's (2001) threshold regression model, which included lagged inflation and investment. According to their findings, Nigeria's optimal inflation rate for growth was 6.0 percent. The illustration of Mathematical Model and Real data from 1996 to 2019, were navigated to evaluate the influence and relationship between Tanzania Inflation and the GDP of Rwanda due to strong trade flow and partnership between the two neighbors, here as considered.

2. REAL DATA ANALYSIS FROM 1996-2019 ON INFLATION AND GDP, RWANDA-TANZANIA

The study was basically based on the influence of Inflation of Tanzania on Rwanda Domestic Product (GDP) and Statistic was performed using R. In this section, a Trend of Trade % GDP of Rwanda and Inflation of Tanzania from 1996 to 2019 was illustrated on a graph below:
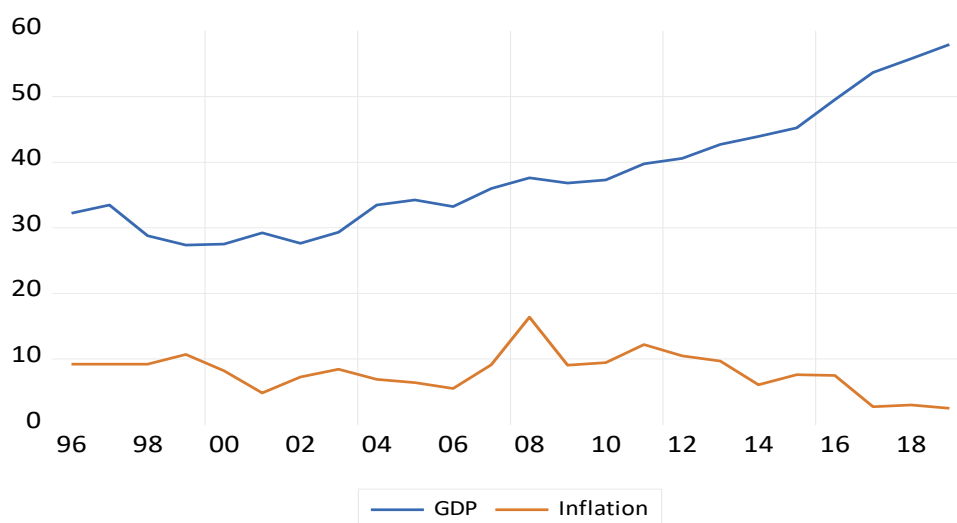

Figure 1: Trend of Trade % GDP of Rwanda and Inflation of Tanzania of 23years

The Figure1 illustrates the trend of both GDP and inflation for Rwanda and Tanzania respectively. According to this figure, trade % GDP increased with time while inflation decreased. From this figure also, it can be said that higher value of inflation corresponds to lower value of GDP.

2.1 Correlation Analysis

In this sub-section, a correlation of the Inflation and GDP between the two countries was considered to check whether there is an association between the two variables and significance.

Correlation t-Statistic Probability	GDP	INFLATION
GDP	1.000000 ----- -----	
INFLATION	-0.444281 -2.326030 0.0296	1.000000 ----- -----

In this table, we investigated the relationship between Trade % GDP of Rwanda and inflation of Tanzania. Based on the table, the correlation coefficient is -0.444 with p-value of 0.0296. This suggests that the two variables are significantly negatively related. This is due to the fact that GDP and inflation are negatively related and that trade between the two countries is strong.

2.2 Regression Analysis

After getting a significant correlation coefficient, the next step would be to estimate a regression model between the two variables. This sub-section was considered to illustrate the linear relationship between the two variables, that is, Inflation of Tanzania and Gross Domestic Product of Rwanda. The illustration is below:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INFLATION	-1.298750	0.558355	-2.326030	0.0296
C	48.40518	4.763348	10.16201	0.0000
R-squared	0.197385	Mean dependent var		38.05295
Adjusted R-squared	0.160903	S.D. dependent var		9.078636
S.E. of regression	8.316234	Akaike info criterion		7.153951
Sum squared resid	1521.514	Schwarz criterion		7.252123
Log likelihood	-83.84742	Hannan-Quinn criter.		7.179996
F-statistic	5.410417	Durbin-Watson stat		0.289254
Prob(F-statistic)	0.029628			

From the results, the coefficient of Inflation is -1.29 with a p-value of 0.029 which is less than 0.05. The negative sign implies that inflation of Tanzania has negative impact on trade % GDP of Rwanda and this confirms with the results obtained from correlation analysis. From this coefficient, it can be said that one-unit increase in Inflation of Tanzania decreases Trade % GDP of Rwanda by 1.29. The estimated model can be written as

$$GDP = -1.29875026955 * INFLATION + 48.4051791926.$$

3. MATHEMATICAL MODEL

In this section, we formulated a Mathematical Model of the influence of Tanzania inflation on Rwanda's Gross Domestic Product (GDP). In this context, we have two probabilities to consider, those are, the Inflation of Tanzania can affect Rwanda's GDP negatively and/or positively if it goes up and/or down; respectively. A Mathematical Model diagram is formulated to understand this scenario, with two compartments and possible outcomes testing, accordingly. The dynamic

of the proposed Model is analyzed by the use of non-linear system of equation derived from it. We consider that the Inflation in general can change to go up and/or down with respect to time t and GDP_R. Let denote the general influence of Tanzania inflation, changes with respect to time, on Rwanda Gross Domestic Product $G(t)$. This comes to the following equation:

$$G(t) = GDP_R(t) + I_T(t)$$

The diagram below is used to illustrate the aforementioned dynamism.
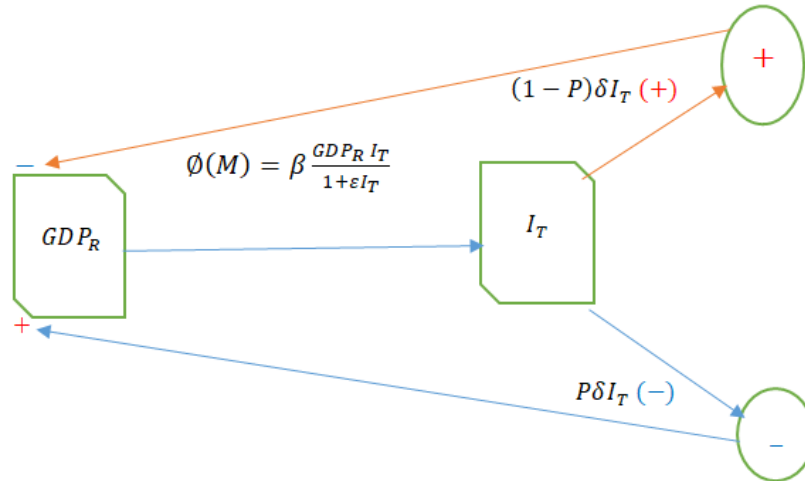

Figure 2: Schematic diagram of the proposed Model.

Variable and parameter description of the proposed Mathematical Model (1)

Variable	Description
GDP_R	Gross Domestic Product of Rwanda
I_T	Inflation of Tanzania
Parameter	**Description**
$\emptyset(M)$	The inflation function
β	The rate of GDP_R being influenced by I_T
ε	The factor at which I_T vary
P	Portion of increase and/or decrease in GDP_R
δ	Contribution rate of which I_T changes, has on GDP_R

Table 1: Variables and parameters of the model.

From Figure 1, we derived a system of non-linear differential equations as shown below:

$$\begin{cases} \dfrac{dGDP_R(t)}{dt} = -\beta \dfrac{GDP_R\, I_T}{1 + \varepsilon I_T} + P\delta I_T - (1-P)\delta I_T \\ \dfrac{d\, I_T(t)}{dt} = \beta \dfrac{GDP_R\, I_T}{1 + \varepsilon I_T} - (1-P)\delta I_T - P\delta I_T \end{cases} \qquad (1)$$

Subjected to the initial conditions: $GDP_R(0) > 0;\ I_T(0) > 0$

3.1 Fundamental properties

In this subsection, we find the fundamental properties of the system (1), which is essential in the next sections of this study.

3.1.1 Positivity of the system with relevance to the study

The associated parameters of the system (1) with respect to the initial conditions are positive for all $t > 0$ and can be proved below.

3.1.2 Theorem.
If $\{GDP_R(0), I_T(0)\}$ and all the associated parameters of the system (1) are positive, then solutions $GDP_R(t), I_T(t)$ are all positive $\forall t > 0$.

Proof: *Consider $t_1 = Sup\{t > 0 : GDP_R(t) \geq 0, I_T(t) > 0\}$*

From the first equation of the system (1),

$$\frac{dGDP_R(t)}{dt} = -\beta \frac{GDP_R\, I_T}{1 + \varepsilon I_T} + P\delta I_T - (1 - P)\delta I_T$$

Since $P\delta I_T - (1 - P)\delta I_T > 0$, then the following equation holds:

$$\frac{dGDP_R(t)}{dt} = -\beta \frac{GDP_R\, I_T}{1 + \varepsilon I_T} - (1 - P)\delta I_T$$

$$\frac{dGDP_R(t)}{dt} = -\left[\beta \frac{GDP_R\, I_T}{1 + \varepsilon I_T}\right]$$

Let

$$\varphi = \frac{\beta I_T}{1 + \varepsilon I_T}$$

$$\frac{dGDP_R(t)}{dt} = -\varphi GDP_R$$

$$\int_0^{t_1} \frac{dGDP_R(t)}{GDP_R} = -\int_0^{t_1} \varphi dt$$

$$ln|GDP_R(t)||_0^{t_1} = -\varphi t_1$$

$$|GDP_R(t)||_0^{t_1} = e^{-\varphi t_1}$$

$$GDP_R(t_1) = GDP_R(0) + e^{-\varphi t_1} > 0.$$

Similarly, the same procedures apply on the second equation of the system (1). Proof can be illustrated below:

$$\frac{d\, I_T(t)}{dt} = \beta \frac{GDP_R\, I_T}{1 + \varepsilon I_T} - (1 - P)\delta I_T - P\delta I_T$$

Since $\beta \frac{GDP_R\, I_T}{1 + \varepsilon I_T} > 0$, the following equation holds:

$$\frac{d\, I_T(t)}{dt} = -1$$

$$\int_0^{t_1} d\, I_T(t) = -\int_0^{t_1} dt$$

$$I_T(t_1) = t_1 < 0.$$

The negative sign indicates the decrease in inflation and this comes, by contrast, to the positive effect on Rwanda Domestic Product with respect to time. The above equation can be rewritten as:

$$I_T(t_1) = t > 0. \forall t > 0$$

From the general influence of Tanzania inflation, changes with respect to time, on Rwanda GDP

$$G(t) = GDP_R(t) + I_T(t)$$

$$\frac{dG(t)}{dt} = \frac{dGDP_R(t)}{dt} + \frac{d\,I_T(t)}{dt} \tag{2}$$

For $\frac{dG(t)}{dt} = 0$, the following equality holds; that is,

$$-\beta\frac{GDP_R\,I_T}{1+\varepsilon I_T} + P\delta I_T - (1-P)\delta I_T + \beta\frac{GDP_R\,I_T}{1+\varepsilon I_T} - (1-P)\delta I_T - P\delta I_T = 0$$

Therefore,

$$-(1-P)\delta I_T - (1-P)\delta I_T = 0$$

$$-2[(1-P)\delta I_T] = 0$$

$$2 - 2P = 0$$

$$P = 1$$

3.2. Numerical simulation of the Model

In this section, we simulate the proposed Mathematical Model (1) using variable and parameter values. We used Python 3.8 programming language to simulate the aforementioned Model. Having the two compartmental variables, and by setting up the initial conditions and parameter values, the following table shows the estimates of the proposed Model.

Variable and parameter values of the proposed Mathematical Model (1)

Variable	Value	Source
GDP_R	60	Estimated
I_T	2	Estimated
Parameter	**Value**	
β	0.00015	Estimated
ε	0.2	Estimated
P	1	Calculated
δ	0.5	Estimated

Table 2: Variables and parameters values of the model.

3.3. Results

The study aim was to investigate whether the inflation of Tanzania has an influence on Rwanda's Gross Domestic Product (GDP). The following Figures were generated from parameter values and variable estimates of the study.
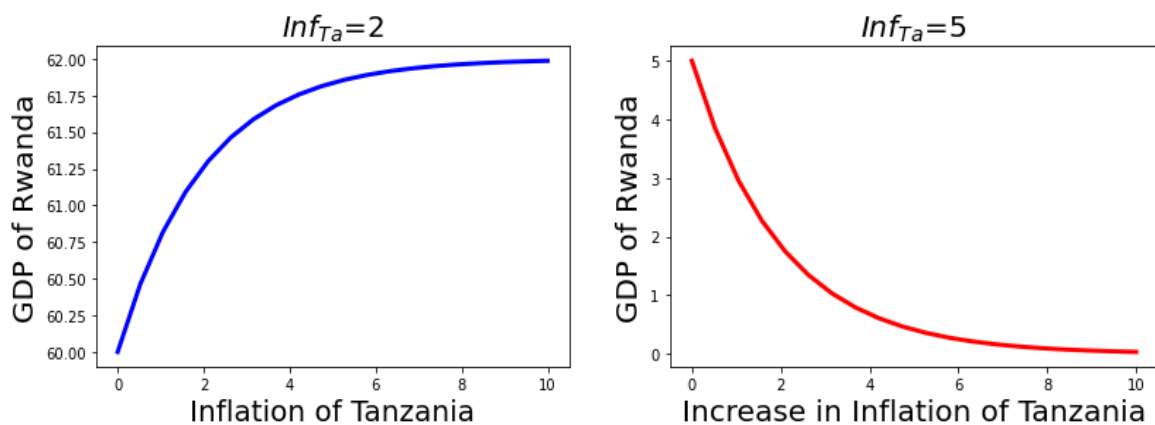

Figure 3(a,b): The effect of Tanzania Inflation on Rwanda's Economy

The Figure 3 (a) show the influence of Tanzania Inflation on Rwanda's Economy on the variable values of 60% Gross Domestic Products (GDP) at the inflation rate of 2, and 0.00015 the rate of Rwanda's Gross Domestic Product being influenced by Inflation of Tanzania. Moreover, the factor at which Inflation of Tanzania vary with respect to time t was estimated to be 0.2, and 0.5 the contribution rate of which Inflation of Tanzania changes, has on Rwanda's Economy. Thus, we calculated the portion of the increase and/or decrease in Rwanda's Domestic Product due to changes in Inflation of Tanzania, that is, $P = 1$. The findings on Figure 3 (a) show that Inflation rate of Tanzania has a positive contribution on Rwanda GDP. However, Figure 3(b) suggests that if the Inflation of Tanzania goes high, the GDP of Rwanda can decrease with respect time t.

3.4 Conclusion and Recommendation

The study objective was to find out whether the decrease and/or increase in Inflation rate of Tanzania can influence GDP of Rwanda. Mathematical Model was used and Simulated Using Python 3.8 Programming language. Statistical Analysis techniques, say, correlation and regression Analysis; also were considered to Analyze the real data of Inflation and GDP between Rwanda and Tanzania for the period of 23years from 1996-2019, using R. The findings for both models give the positive results. The more inflation rate of Tanzania decreases, the more GDP of Rwanda increases. The two models indicated that, as long as Inflation of Tanzania goes high, the GDP of Rwanda would fall down. The government of Rwanda is recommended to strengthen and kept forward the trade flow with Tanzania as it has a positive impact on Rwanda GDP.

REFERENCES

[1] Ayyoub, M., Chaudhry, I. S., & Farooq, F. (2011). Does Inflation Affect Economic Growth? The case of Pakistan. *Pakistan Journal of Social Sciences (PJSS)*, *31*(1).

[2] Khan, M. S., & Senhadji, A. (2000). Threshold effects in the relationship between inflation and growth.

[3] Fabayo, J. A., & Ajilore, O. T. (2006). Inflation: How much is too much for economic growth in Nigeria. *Indian Economic Review*, 129-147.

[4] Gonzalo, J., & Pitarakis, J. Y. (2002). Estimation and model selection based inference in single and multiple threshold models. *Journal of Econometrics*, *110*(2), 319-352.

[5] Hosny, A., Kandil, M., & Mohtadi, H. (2014). What does Egypt's Revolution Reveal about its Economy?. *International Economic Journal*, *28*(4), 589-611.

[6] Barro, R. J. (1996). Inflation and growth. *Review-Federal Reserve Bank of Saint Louis*, *78*, 153-169.

[7] Bawa, S., & Abdullahi, I. S. (2012). Threshold effect of inflation on economic growth in Nigeria. *CBN Journal of Applied Statistics*, *3*(1), 43-63.

[8] Fischer, S. (1993). The role of macroeconomic factors in growth. *Journal of monetary economics*, *32*(3), 485-512.

[9] Hamilton, A. (2001). Exploding inflation. *Zeal Intelegence*.

[10] Hansen, B. E. (1999). Threshold effects in non-dynamic panels: Estimation, testing, and inference. *Journal of econometrics*, *93*(2), 345-368.

[11] Melberg, H. O. (1992). Inflation: An overview of theories and Solutions. *Retrieved June*, *23*, 2008.

[12] Ojo, M. O. (2000). The Role of the Autonomy of the Central Bank of Nigeria (CBN) in Promoting Macroeconomic Stability. *Central Bank of Nigeria Economic and Financial Review*, *38*(1), 88-101.

[13] Reddy, D. L. (2012). Impact of inflation and GDP on stock market returns in India. *International journal of advanced research in management and social sciences*, *1*(6), 120-136.

[14] Singh, K., & Kalirajan, K. (2003). The inflation-growth nexus in India: an empirical analysis. *Journal of policy modeling*, *25*(4), 377-396.

[15] Semuel, H., & Nurina, S. (2014). *Analysis of the effect of inflation, interest rates, and exchange rates on Gross Domestic Product (GDP) in Indonesia* (Doctoral dissertation, Petra Christian University).